Phase Transition of Message Propagation Speed in Delay Tolerant Vehicular Networks*

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Abstract–Delay tolerant network (DTN) architectures have recently been proposed as a means to enable efficient routing of messages in vehicular area networks (VANETs), which are characterized by alternating periods of connectivity and disconnection. Under such architectures, when multihop connectivity is available, messages propagate at the speed of radio over connected vehicles. On the other hand, when vehicles are disconnected, messages are carried by vehicles and propagate at vehicle speed. Our goal in this paper is to analytically determine what gains are achieved by DTN architectures and under which conditions, using average message propagation speed as the primary metric of interest. We develop an analytical model for a bi-directional linear network of vehicles, as found on highways. We derive both upper and lower bounds on the average message propagation speed, by exploiting a connection with the classical pattern matching problem in probability theory. The bounds reveal an interesting phase transition behavior. Specifically, we find out that below a certain critical threshold, which is a function of the traffic density in each direction, the average message speed is the same as the average vehicle speed, i.e., DTN architectures provide no gain. On the other hand, we determine another threshold above which the average message speed quickly increases as a function of traffic density and approaches radio speed. Based on the bounds, we also develop an approximation model for the average message propagation speed that we validate through numerical simulations.

Keywords: DTN, VANETs, message propagation, vehicular networking, V2V, phase transition.

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1 Introduction

Vehicles equipped with wireless communication technologies are regarded as nodes of a unique network described as a vehicular ad hoc network or VANET. There are several benefits to enabling *messaging*, i.e., the ability of exchanging messages between vehicles. Safety messaging, real-time updates on traffic and congestion along with enabling Internet access are some of the envisioned services [3].

Several architectures have been proposed for inter-connecting vehicles on the roadway. These include infrastructure-based models where vehicles communicate directly with roadside infrastructure, such as access points or cellular towers [4]. Another solution is an ad hoc model where vehicles on the roadway communicate in an ad hoc network supported by multihop networking [5]. An innovative solution adopts a *delay tolerant networking* (DTN) model that exploits opportunistic connectivity between vehicles moving in *opposing directions* to achieve greedy data forwarding [6, 7, 8]. In the absence of connectivity, messages are cached in a vehicle's memory and travel at vehicle's speed. When connectivity is restored, messages are forwarded multihop at radio speed, which is typically at least an order of magnitude larger than the vehicle speed [9].

The main purpose of this work is to analyze and provide quantitative insight into the impact of the VANET environment on the performance of DTN messaging protocols. For instance, vehicles on a roadway often travel at relatively high speeds (e.g., 20 m/s or 72 kmph). Thus, considering bi-directional traffic, the topology of the network potentially changes at a fast rate. Another important factor is vehicle density on the roadway. Vehicle traffic density varies depending upon the type of roadway (rural/urban) and time of the day (night/day). Traffic densities of 10 vehicles/km are considered low traffic volumes, $25-40$ vehicles/km are considered medium traffic densities, while densities of > 60 vehicles/km are considered high [7, 10].

In this work, we develop an analytical model to characterize the average propagation speed of messages over a long distance in a delay tolerant network formed over moving vehicles. The model can be applied to unicast, broadcast or multicast applications. Through the course of our analysis, we determine how radio and network parameters, such as the radio range, speed of vehicles, and traffic density in both directions, influence the average message propagation speed. Our model captures the dynamic behavior of the network connectivity graph, as a result of vehicular mobility.

In this context, the main contributions of this paper are the following. First, we develop an analytical model for message propagation in a dynamic network formed over vehicles traveling in opposing directions and characterized by transient connectivity. The model captures the random nature of distance between vehicles. Under such a model, we derive *upper* and *lower* bounds on the average message propagation speed. Throughout our analysis, we establish a relationship with the classical *pattern matching* problem in probability theory [11]. We exploit this relationship to compute upper and lower bounds on the average distance traversed during periods of disconnection.

Based on the analysis, our second main contribution is to establish the existence of a phase transition in the properties of message propagation in the network as a function of the density of vehicles in the network. The phase transition is important as it reveals different regimes in which DTN architectures help or not in improving performance. Specifically, we find out that below a certain critical threshold, which is a function of the traffic density in each direction, the average message speed is the same as the average vehicle speed, i.e., DTN architectures provide no gain. On the other hand, we determine another threshold above which the average message speed quickly increases as a function of traffic density and approaches radio speed.

Last, we use the analytical model to develop a simple approximation on the average message propagation speed. We validate this approximation, through various simulations run with different network parameters. This approximation model provides means for quick evaluation of VANET performance, without the need of running lengthy simulations.

The rest of the article is organized as follows. Section 2 describes related work. Section 3 details the vehicular networking environment and relevant observations on how messages propagate in DTNs. In Section 4, we present a detailed description of our analytical model, derive bounds and approximation on the average message propagation speed, and establish the phase transition behavior. Simulation results are compared with the bounds and the approximation model in Section 5. We conclude the paper in Section 6 with a discussion of the results.

2 Related Work

In the context of vehicular networks, DTN messaging has been proposed in previous work in [7, 8, 9, 12, 6, 13]. In reference [7], the authors have evaluated vehicle traces on the highway and demonstrated that they closely follow exponential distribution of nodes. The work demonstrates network fragmentation and the impact of time varying vehicular traffic density on connectivity and hence, the performance of message propagation. The UMass DieselNET project explores the deployment of communication infrastructure over campus transportation network and records measurements on opportunistic networking [14].

Several works have developed analytical models studying message propagation in VANETs. In reference [15], the authors study in detail the propagation of critical warning messages in a vehicular network. The authors develop an analytical model to compute the average delay in delivery of warning messages as a function of vehicular traffic density. Our work is unique in that we consider data propagation in the event of a partitioned network. However, our model is consistent with this work with respect to the network assumptions, e.g., exponential distribution of nodes in a one-dimensional highway setting. Another model proposed in [16], assumes exponential distribution of nodes to study connectivity based on queueing theory. The authors describe the effect of system parameters such as speed distribution and traffic flow to analyze the impact on connectivity. However, the authors do not consider a store and forward mechanism from which gains can be achieved.

Wu *et al.* have proposed an analytical model to represent a highway-vehicle scenario [9]. In their approach, they investigate speed differential between vehicles traveling in the same direction to bridge partitioned network of vehicles. They also provide analysis for the case where vehicles in the opposing direction are used for propagating messages, similar to our approach. Yet, their results are less explicit than ours due to the higher complexity of their model. In [17] and [18], the authors also propose to use opposing traffic to bridge connectivity. They refer to this technique as *transversal message hopping*, as opposed to *longitudinal message hopping* which exploits traffic of vehicles in the same direction for messaging. They compute the distribution of the latency of communication between two cars located at a given distance, using either of these two techniques. In contrast, our DTN messaging scheme, described in the next section, achieves significant performance gain by combining both the longitudinal and traversal techniques. The analysis of such a mixed scheme is more involved. Moreover, the phase transition phenomenon revealed by this analysis is a distinct contribution of our work.

Phase transition phenomenon in the context of ad hoc networks has been discussed in reference [19]. The authors discuss a model of random placement of nodes in a unit disk and analyze the probabilistic properties of the connectivity graph in the context of increasing communication radius. In reference [20], authors study the availability of transient paths of short hop-length in a mobile network and observe that a phase transition occurs as time and hops are jointly increased according to the logarithm of the network size. Authors in [21] have studied information dissemination in a network with unreliable links. Several works have studied connectivity characteristics in a one-dimensional linear arrangement of nodes [22], [23], [24]. Our work is unique in that it considers a linear arrangement of nodes that are *mobile* in *opposing* directions as compared to existing models that consider static networks. Our transient connectivity and delay tolerance assumptions are unique and distinct from previous work. In reference [25], authors have demonstrated that mobility increases the capacity of an ad hoc wireless network. An analytical model developed by the authors demonstrates that for one-dimensional and random mobility patterns the interference decreases and often mobility aids in improving network capacity. In a similar context, we demonstrate that under certain conditions on traffic density, increased mobility aids in speeding-up message propagation.

Preliminary findings leading to this work were presented in [1, 2]. The work in [1] presented preliminary analysis on the average message propagation speed. It did not elaborate on the phase transition phenomenon and did not include an approximation on the average message propagation speed. The work in [2] assumed a different model on the inter-vehicular distance, i.e., fixed distance between nodes in one direction of the highway.

3 Vehicular Networking Environment

A network formed over moving vehicles has characteristics of topology and mobility that are distinct from traditional mobile ad hoc networks. In this section, we describe key observations and assumptions of the vehicular networking environment. We describe the highway environment, the nature of vehicle mobility and the time-varying density of vehicular traffic. We discuss the impact of these observations on the message exchange. Based on these observations, we describe a delay-tolerant messaging scheme that exploits opportunistic connectivity between nodes to forward data. The messaging scheme forms the basis of our analytical model. We describe the sequence of events in message propagation as the network transitions between states of connectivity and disconnection.

3.1 Highway Model

We consider a highway scenario where vehicles travel in either direction on a bi-directional roadway. We assume that vehicles are equipped with storage, computation and communication capabilities. The roadway is annotated as *eastbound* and *westbound* for convenience in the narrative. The highway model is illustrated in Figure 1. We assume that vehicles travel in both directions. In this work, we consider a single lane on each side of the highway. However, our model could apply to scenarios of multiple lanes as well. The traffic in each lane can be each modeled as an independent Poisson process. If vehicle move at the same speed on each lane, then the arrivals can be combined to form a single Poisson process.

A fixed radio range model is assumed such that vehicles within range are able to communicate with each other (Ref. [26] describes a practical method for estimating the communication range). As vehicles travel on the roadway, the topology of the network changes, nodes come in intermittent contact with vehicles traveling in *opposing* directions. These opportunistic contacts can be utilized to aid message propagation, as explained in subsequent text.

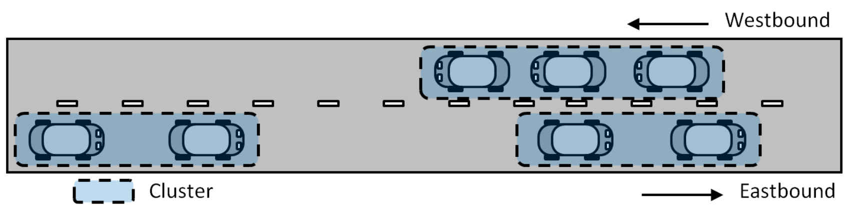


Figure 1: Illustration of the highway model and clustering of vehicles on the roadway.

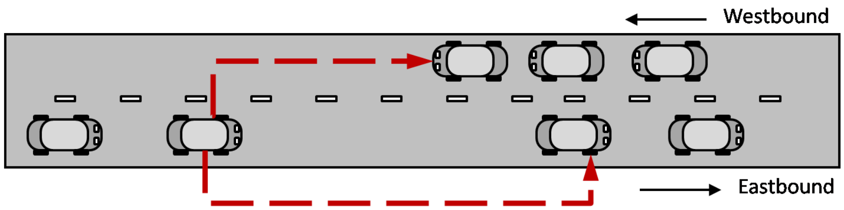
3.2 Network Partitioning

Vehicle traffic density on the roadway is a time-varying quantity. Road traffic statistics and time-series snapshots of vehicular traffic have demonstrated that vehicles tend to travel in clusters on the roadway [27]. The clusters tend to be separated by some distance. Thus, in networking terms, the network is *partitioned*, i.e., the network is composed of disconnected sub-nets that are partitioned from each other, illustrated in Fig. 1. However, the network topology changes as vehicles travel in opposing directions. Sub-nets come in intermittent contact with other sub-nets. Thus, sub-nets connect and disconnect frequently leading to time-varying partitioning.

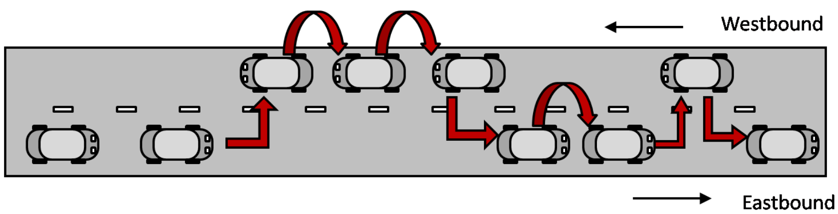
In a network formed over moving vehicles, enabling messaging is challenging due to the absence of a fully connected network. The network is sparsely populated and there is lack of end-to-end connectivity in the network. MANET schemes that rely on end-to-end connectivity are a poor solution as a path from source to destination may not exist due to lack of sufficient node density in the network. Even if vehicle traffic traveling in opposing directions is included in path formation, the resulting paths are short-lived. Thus, routing schemes based on path formation strategies are an inefficient solution as a result of the increased overhead involved in path formation and path maintenance. Thus, the requirement is of a messaging scheme that is able to adapt to the extremes of a sparse and dense node density and, at the same time, solve the problem of partitioning.

3.3 Messaging Model

In a related work [8], we propose a messaging scheme that enables us to solve the problems of network partitioning. A brief description of the scheme is provided here. The scheme relies on source and destination pairs identified on the basis of location. A common assumption in the VANET environment is GPS equipped vehicles that are location aware and share this information in a neighborhood. We propose to exploit the spatial-temporal correlation of data and nodes in the system. The data are identified as sourced from a location and destined for a location. The location coordinates obtained from GPS are embedded in each packet such that each packet is attributed (labelled). Thus, we are able to implement a simplified geographic routing protocol as each intermediate node forwards data based on its location and the source-destination locations embedded in the data packets. The scheme does not require the formation of an end-to-end path, rather each node is able to route based on the attributed data.



(a) At $t = 0$, the network is partitioned and nodes are unable to communicate.



(b) At $t = \Delta t$, topology changes, connectivity is achieved and vehicles are able to communicate.

Figure 2: Illustrating delay tolerant network (DTN) messaging as the network connectivity changes with time.

While the time-varying connectivity in the network presents a challenge to enable networking, it provides an opportunity to bridge the partitioning in the network. As vehicles traveling in one direction are likely to be partitioned, vehicles that are traveling in the opposing direction can be used as illustrated in Fig. 2(b). This transient connectivity can be used irrespective of the direction of data transfer, *eastbound* or *westbound*.

However, it is important to note that this connectivity is not always instantaneously available. Partitions exist on either side of the roadway and in a sparse network there are large gaps between connected sub-nets. Here we propose the application of delay tolerant networking (DTN) [28], [29]. DTN is essentially a *store-carry-forward* scheme where messages are cached or buffered in a node's memory when the network is disconnected. The data are forwarded as and when connectivity is available in the system. This is illustrated in Fig. 2, where at the time of reference $t = 0$, the network is partitioned and there is lack of instantaneous connectivity between nodes. At time instant $t = \Delta t$, the topology of the network changes by virtue of vehicle mobility and connectivity between previously partitioned nodes is available.

The message propagation is a function of the connectivity graph formed over vehicles. Consider a message propagation goal in the eastbound direction. The message originating at a vehicle encounters a partition, as shown in Fig. 2(a). As the network is partitioned, the message is cached within a node's memory. As the vehicle traverses some distance, the topology of the network changes. Connectivity is sought over westbound nodes as the eastbound nodes are partitioned. For connectivity to the next eastbound node, there should be sufficient density of nodes along westbound to bridge the partition. Once connectivity is achieved, the messages are able to propagate multihop over connected nodes in either eastbound or westbound direction until the next partition is encountered. Thus, the message propagation alternates between periods of multihop propagation and disconnection. In the next section, we compute, analytically, bounds on the expectations of the time periods during which the network is connected or disconnected, as a function of the traffic density in the eastbound and westbound directions. Hence, we can characterize the average speed at which messages propagate in the network.

4 Analysis

In the previous section, we described and identified the challenges that lie in enabling inter-vehicle communication. We outlined the highway model of a vehicular area network. The partition observed in the network is solved by using a unique messaging model that applies techniques from delay tolerant networking (DTN) to achieve opportunistic and greedy data forwarding. Our goal henceforth in this paper is to characterize the average speed of message propagation in such a delay tolerant network formed over moving vehicles. In this section, we introduce an analytical model and derive bounds on the message propagation speed averaged over time revealing a phase transition behavior. We also provide an approximation model following the same lines as the derivation of the bounds.

4.1 Model and Notation

We consider a bi-directional roadway scenario wherein vehicles travel in either *eastbound* or *westbound* directions, as illustrated in Fig. 1. Vehicles are assumed to be point objects such that the length of a vehicles is not taken into account while computing distance. The model is a linear one-dimensional approximation of the roadway absent any infrastructure, such that vehicles form nodes of a linear ad hoc network. In each direction, nodes are assumed to move at a constant speed v m/s such that the distance between nodes moving along the same direction remains unchanged. We assume a fixed transmission range R. Thus, two nodes are directly connected by a radio link if the distance between them is R or less. The distance X between any two consecutive nodes is an i.i.d. exponential random variable, with parameter λ_e for *eastbound* traffic and λ_w for *westbound* traffic. The exponential distribution has been shown to be in good agreement with real vehicular traces under uncongested traffic conditions [7]. Our work focuses on that particular scenario, where as vehicular traffic moves in opposing directions, periods of connectivity alternate with periods of disconnection. As such, the primary metric of interest in this paper is the *average message propagation speed* (v_{avg}), a quantity measured between two distant points on the road, using the side of the road as the frame of reference

Without loss of generality, we will focus in the sequel on computing the average message propagation speed in the *eastbound* direction. The *westbound* average propagation speed can be found by simply substituting east and west indices in all the formulae. Once v_{avg} is derived, one can easily compute the average message propagation speed with respect to a vehicle moving at speed v, by changing the frame of reference from the road side to that of the vehicle. Thus, from the perspective of a vehicle, the average propagation speed of a message sent to it from a vehicle located far behind it is $v_{avg} - v$. If the message is sent from a vehicle located far ahead, the average speed is $v_{avg} + v$. The source can be either on the same lane or on the opposing lane, since initial conditions do not affect long-term average performance.

We refer to the alternating periods of disconnection and (multihop) connectivity as *phase 1* and *phase 2*, respectively. In phase 1, when nodes are disconnected, by the assumption of delay tolerance, data messages are buffered at nodes until connectivity becomes available through a subset of nodes moving in the opposing direction. The messages traverse a physical distance as the vehicle travels at speed v m/s, waiting for connectivity to be renewed. In phase 2, when multihop connectivity is available, data propagate at radio speed v_{radio}. Connectivity is maintained as long as consecutive nodes traveling in a given direction are located at distance smaller than R or if subnet of nodes moving in the opposing direction can bridge the partition between the nodes. The multihop radio propagation speed is determined by characteristics of the physical and network layers. It is typically at least an order of magnitude larger than the vehicle speed, i.e. $v_{radio} >> v$. A typical value is $v_{radio} = 1000$ m/s, as obtained from measurements [9]. The average message propagation speed v_{avg} is a function of the time spent in the two alternating phases.

A *cycle* is defined as a phase 1 period followed by a phase 2 period. Denote by T_1^n and T_2^n the random amounts of time a message spends in the two phases, during the n-th cycle, where $n = 1, 2, \ldots$. The random vectors $(T_1^n, T_2^n), n \geq 1$ are i.i.d., due to the memoryless assumption on the inter-vehicular distances. Note, however, that T_1^n and T_2^n are not independent. Indeed, both T_1^n and T_2^n depend on the distance between the vehicle carrying the message at the beginning of cycle n and the next vehicle traveling in the same direction.

Based on our statistical assumptions, the system can be modeled as an *alternating renewal process* [11], where message propagation cyclically alternates between phases 1 and 2. Denote $E[T_1] = E[T_1^n]$ the expected time spent in phase 1 and $E[T_2] = E[T_2^n]$ the expected time spent in phase 2. Then, the long-run fraction of time spent in each of these states is respectively [11]:

$$p_1 = \frac{E[T_1]}{E[T_1] + E[T_2]}; \qquad p_2 = \frac{E[T_2]}{E[T_1] + E[T_2]}. \tag{1}$$

Given that the average time spent in phase 1 and phase 2 are $E[T_1]$ and $E[T_2]$ respectively, while the rate of propagation in each phase is v m/s and v_{radio} m/s respectively, we can compute the average message propagation speed v_{avg} as follows:

$$\begin{aligned} v_{avg} &= p_1 v + p_2 v_{radio} && (2)\\ &= \frac{E[T_1]v + E[T_2]v_{radio}}{E[T_1] + E[T_2]} && (3)\\ &= \frac{E[D_1] + E[D_2]}{E[D_1]/v + E[D_2]/v_{radio}}, && (4) \end{aligned}$$

where $E[D_1]$ and $E[D_2]$ are the expected distances traversed by a message in phase 1 and phase 2 of a cycle.

The primary goal of our analysis is to determine how $E[D_1]$ and $E[D_2]$ (and thereby the average message propagation speed v_{avg}) depend on the parameters λ_e, λ_w, R, v, and v_{radio}. Since the derivation of exact expressions for these quantities is difficult, we introduce next a discretization of the system allowing to compute upper and lower bound on the average message propagation speed when $v_{radio} = \infty$. Note that in that case:

$$v_{avg} = \left(1 + \frac{E[D_2]}{E[D_1]}\right) v. \tag{5}$$

4.2 Discretization

The analysis of the problem at hand is rendered difficult by its continuous nature. Specifically, if the distance between two nodes traveling in a given direction exceeds R, determining the probability that the nodes are connected through nodes traveling in the opposing direction is a difficult combinatorial problem. To circumvent this difficulty, we discretize the roadway into cells, each of size l. In the sequel, we discuss how to select appropriate values of l for the derivation of upper and lower bounds.

We consider a cell to be *occupied* if one or more vehicles are positioned within that cell. By virtue of the memoryless property of the exponential distribution, the probability p that a cell is occupied is $p = (1 - e^{-\lambda l})$, where l is the cell size and λ is the traffic density. For cells along the *eastbound* direction, the probability that a cell is occupied is $p_e = (1 - e^{-\lambda_e l})$, whereas for the *westbound* direction it is $p_w = (1 - e^{-\lambda_w l})$.

Upper bound To derive an upper bound on v_{avg}, we set $l = R$. Thus, we require each adjacent cell of length R to be occupied by at least one node as a condition to guarantee connectivity. This is an optimistic view of the system, since in reality, nodes located in adjacent cells may be separated by a distance greater than R, in fact as much as $2R$. Hence, requiring the presence of at least one node in each cell of size R is a necessary but insufficient condition, in general.

In addition, to simplify the analysis, we assume that all nodes located in a cell are located at the far-end extremity of that cell, except for the first cell for which use the exact inter-distance distribution. Again, this provides an optimistic view, since the average distance computed that way between any two consecutive nodes traveling in the same direction is larger than what it is in reality. Note that, due to the cell discretization, it does not affect the probability that two consecutive nodes are connected. The inter-distance distribution between node is expressed with the following mixed probability distribution:

$$\begin{aligned} f_{X_u}(x) =& \lambda e^{-\lambda x}((u(x) - u(x-R)) \\ & + \sum_{n=1}^{\infty}(e^{-\lambda n R} - e^{-\lambda(n+1)R})\delta(x-(n+1)R), \\ & \text{for } x \geq 0, \end{aligned} \tag{6}$$

where $u(x)$ is the unit step function and $\delta(x)$ is the Dirac delta function [30]. The quantity X_u denotes a random variable distributed according to the upper bound distribution of the inter-vehicle distance.

Thus, for the first cell, the inter-vehicle distance distribution between two nodes is exact and described by the original exponential distribution. However, when $x > R$ for each successive cell, we assume that nodes are located at the far-end extremity of the cell. With the nodes assumed to be placed at the end of each cell, the distance at each iteration becomes a fixed quantity and, hence, easier to compute. Thus, any node located in the second cell, i.e., at a distance between R and $2R$ from the preceding node, is assumed to be located at $2R$. The message propagation distance is then computed as $2R$, and so forth for the next cells.

$$E[D_1]_u = \begin{cases} \frac{R(1-e^{\lambda_e R})}{2\Pr(\bar{C}_u)} \Big[\frac{1}{e^{-\lambda_w R}} \Big\{ \frac{e^{-\lambda_e R}}{1-e^{-\lambda_w R}-e^{-\lambda_e R}} + \frac{(1-e^{-\lambda_w R})e^{-\lambda_e R}}{1-e^{-\lambda_e R}(1-e^{-\lambda_w R})} \\ -\frac{2e^{-\lambda_e R}}{1-e^{-\lambda_e R}} \Big\} - \Big\{ \frac{e^{-\lambda_e R}}{(1-e^{-\lambda_e R})^2} - \frac{e^{-\lambda_e R}(1-e^{-\lambda_w R})}{(1-e^{-\lambda_e R}(1-e^{-\lambda_w R}))^2} \Big\} \Big] & \text{if } e^{-\lambda_e R} + e^{-\lambda_w R} < 1 \\ \infty & \text{otherwise.} \end{cases} \tag{8}$$

Lower bound To derive a lower bound on v_{avg}, we set $l = R/2$. Indeed, when the cell size is $R/2$, nodes in adjacent cells are surely connected, irrespective of their location within their cells. Thus, even for nodes located at the two extremes of adjacent cells, the maximum distance between them is R, which is within communication range. Thus, for the lower bound, we set as a condition for connectivity that each adjacent cell of length $R/2$ be occupied by at least one node. Clearly, it is a sufficient condition, though not always necessary (i.e., two nodes may be connected even if the cell between them is empty).

Similar to Eq. (6), we assume that the distribution of nodes located at a distance smaller than R is the same as the original exponential distribution, while for each subsequent cell of size $R/2$, we assume that the nodes are placed at the near-end extremity of each cell. Thus, we arrive at the following conservative estimate on the probability distribution of the distance:

$$\begin{aligned} f_{X_l}(x) =& \lambda e^{-\lambda x}((u(x) - u(x-R)) \\ &+ \sum_{n=1}^{\infty} (e^{-\lambda(n+1)\frac{R}{2}} - e^{-\lambda(n+2)\frac{R}{2}}) \delta(x - (n+1)\frac{R}{2}), \\ & \text{for } x \geq 0. \end{aligned} \tag{7}$$

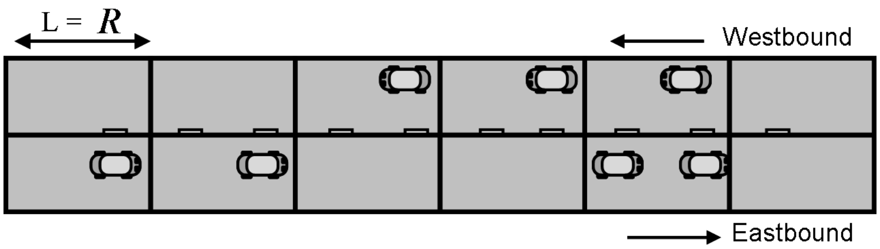
Here, X_l is a random variable following the lower bound distribution of inter-vehicle distance. Figure 3 illustrates the lower and upper bounds.

4.3 Relationship with Pattern Matching Problem

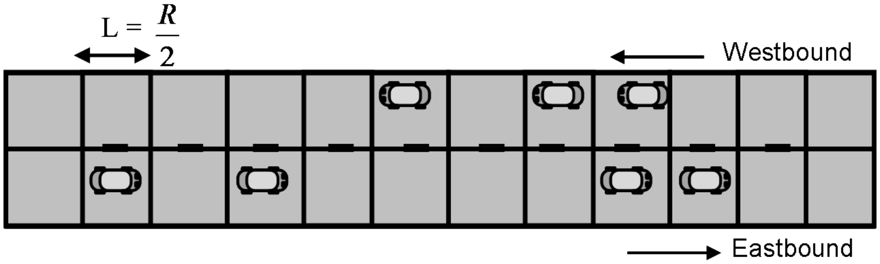
If the distance between two *eastbound* nodes is greater than R, then connectivity must be achieved using nodes along *westbound* direction. As per the discretization described above, the distance is equivalent to, say, N cells. Assuming $v_{radio} = \infty$, the nodes along *eastbound* are connected if each of the N westbound cells in the gap is occupied by at least one node, an event which occurs with probability $(p_w)^N = (1 - e^{-\lambda_w l})^N$.

In the event that not all of the N cells in the *westbound* direction are occupied, the nodes along *eastbound* are deemed to be disconnected. A message is buffered in the node's cache until connectivity is achieved again. The node and, hence, the message traverse some distance (cells) until connectivity is achieved. The number of cells traversed until connectivity is achieved is analogous to the number of trials until a sequence is seen. This is described as *pattern matching* in classical probability theory [11]. The pattern matching problem describes the task to compute the expected number of trials Y until N consecutive successes are obtained, which is given by the relation:

$$E[Y] = \frac{1 - p^N}{(1-p)p^N}, \tag{9}$$



(a) Upper bound: With $l = R$, necessary but insufficient condition.



(b) Lower bound: With $l = R/2$, sufficient but not always necessary condition.

Figure 3: Illustrating the discretization of node distribution on the roadway, *upper* and *lower* bounds for connectivity.

where p is the probability of success in a trial. This is analogous to our problem as we try to find the number of cells traversed by a node until N consecutive cells along *westbound* traffic are occupied by one or more nodes. We exploit this analogy for our analysis in the next section.

4.4 Upper Bound Analysis

In this section, we derive an upper bound on the average message propagation speed v_{avg}, based on the discretized system described in Section 4.2, i.e., assuming cells of size R and an inter-node distance distribution as given by Eq. (6). We denote by $E[D_1]_u$ and $E[D_2]_u$ the expected distances traversed by a message in phase 1 and phase 2 during each cycle. Once these quantities are computed, an upper bound on the average message propagation speed v_{avg} follows readily from Eq. (5). The following Lemma provides an expression for $E[D_1]_u$.

Lemma 4.1 *The expectation of the distance traversed in phase 1 in the upper bound system* $E[D_1]_u$ *is given by Eq.* (8)*, where* $\Pr(\bar{C}_u)$ *is the probability that two consecutive eastbound nodes are disconnected, the expression of which is given by Eq.* (18)*.*

Proof: In phase 1, two consecutive *eastbound* nodes are disconnected from each other. Thus, there is a gap of $N \geq 1$ cells between the nodes, where N is discrete random variable. To bridge this gap, N cells along the *westbound* direction must each be occupied by at least one node. The data are cached in the first node's memory until connectivity is achieved. Owing to node mobility, a physical distance is covered in this time delay. The expected number of cells traversed until connectivity over *westbound* cells is achieved is as given in Eq. (9). Note, however, that the last N cells are traversed at speed v_{radio}, and therefore, should be accounted as part of phase 2 rather than

$$
\begin{aligned}
E[D_1]_u &= \sum_{n=1}^{\infty} E[D_1|N=n]_u \Pr(N=n|\bar{C}_u) \\
&= \frac{R}{2\Pr(\bar{C}_u)} \sum_{n=1}^{\infty} \left[\frac{1-(1-e^{-\lambda_w R})^n}{(e^{-\lambda_w R})(1-e^{-\lambda_w R})^n} - n\right]\left[(1-(1-e^{-\lambda_w R})^n)(e^{-\lambda_e nR} - e^{-\lambda_e (n+1)R})\right]. \quad (14)
\end{aligned}
$$

$$
\begin{aligned}
E[D_2]_u = & \frac{R(1-e^{-\lambda_e R})}{\Pr(\bar{C}_u)}\left[\frac{e^{-\lambda_e R}}{(1-e^{-\lambda_e R})} + \frac{e^{-\lambda_e R}}{(1-e^{-\lambda_e R})^2} - \frac{e^{-\lambda_e R}(1-e^{-\lambda_w R})}{1-e^{-\lambda_e R}(1-e^{-\lambda_w R})} - \frac{e^{-\lambda_e R}(1-e^{-\lambda_w R})}{(1-e^{-\lambda_e R}(1-e^{-\lambda_w R}))^2}\right] \\
& + \frac{1}{\Pr(\bar{C}_u)}\left[\frac{1}{\lambda_e}\left[1-e^{-\lambda_e R}(1+\lambda_e R)\right] + R(1-e^{-\lambda_e R})\left[\frac{e^{-\lambda_e R}(1-e^{-\lambda_w R})}{1-e^{-\lambda_e R}(1-e^{-\lambda_w R})}\right.\right. \\
& \left.\left. + \frac{e^{-\lambda_e R}(1-e^{-\lambda_w R})}{(1-e^{-\lambda_e R}(1-e^{-\lambda_w R}))^2}\right]\right]. \quad (15)
\end{aligned}
$$

---

phase 1. Hence, we subtract them from the computation. Thus, for a given separation between *eastbound* nodes $N = n$, the expected distance traversed until connectivity is given by:

$$
E[D_1|N=n]_u = \frac{R}{2}\left[\frac{1-(1-e^{-\lambda_w R})^n}{e^{-\lambda_w R}(1-e^{-\lambda_w R})^n} - n\right] \quad (10)
$$

Note that a correction factor of $1/2$ is applied as nodes in either direction, eastbound and westbound, are traveling at v m/s. Thus, the distance traversed until connectivity is effectively halved.

Our next goal is to compute $E[D_1]_u$, i.e., the expected distance traversed in phase 1 without conditioning on the gap size. Denote by $\bar{C}_u$, the event that two consecutive eastbound nodes are disconnected. Then,

$$
E[D_1]_u = \sum_{n=1}^{\infty} E[D_1|N=n]_u \Pr(N=n|\bar{C}_u). \quad (11)
$$

We compute $\Pr(N=n|\bar{C}_u)$ using Bayes' Law, i.e.:

$$
\Pr(N=n|\bar{C}_u) = \frac{\Pr(\bar{C}_u|N=n)\Pr(N=n)}{\Pr(\bar{C}_u)}. \quad (12)
$$

We have

$$
\Pr(\bar{C}_u|N=n) = 1-(1-e^{-\lambda_w R})^n, \quad (13)
$$

which is the probability that two consecutive nodes are disconnected given that the separation between them is n cells. This event occurs if the n cells along the westbound direction are not all occupied. Next, we compute the probability that the separation between consecutive eastbound nodes is n cells. This quantity is given by the expression:

$$
\Pr(N=n) = (e^{-\lambda_e nR} - e^{-\lambda_e (n+1)R}). \quad (17)
$$

$$
\begin{aligned}
E[D_{2,1}]_u &= R\sum_{n=1}^{\infty}(n+1)\Pr\left(N=n|\bar{C}_u\right) \\
&= \frac{R(1-e^{-\lambda_e R})}{\Pr(\bar{C}_u)}\left[\frac{e^{-\lambda_e R}}{(1-e^{-\lambda_e R})}+\frac{e^{-\lambda_e R}}{(1-e^{-\lambda_e R})^2}-\frac{e^{-\lambda_e R}(1-e^{-\lambda_w R})}{1-e^{-\lambda_e R}(1-e^{-\lambda_w R})}-\frac{e^{-\lambda_e R}(1-e^{-\lambda_w R})}{(1-e^{-\lambda_e R}(1-e^{-\lambda_w R}))^2}\right]
\end{aligned}
\tag{16}
$$

Finally, the probability that two nodes are disconnected can be computed as:

$$
\begin{aligned}
\Pr\left(\bar{C}_u\right) &= \sum_{n=1}^{\infty}\Pr\left(\bar{C}_u|N=n\right)\Pr\left(N=n\right) \\
&\text{substituting from Eqs. (13), (17)} \\
&= \sum_{n=1}^{\infty}(1-(1-e^{-\lambda_w R})^n)(e^{-\lambda_e nR}-e^{-\lambda_e(n+1)R}) \\
&= (1-e^{-\lambda_e R})\left[\frac{e^{-\lambda_e R}}{1-e^{-\lambda_e R}}-\frac{e^{-\lambda_e R}(1-e^{-\lambda_w R})}{1-e^{-\lambda_e R}(1-e^{-\lambda_w R})}\right].
\end{aligned}
\tag{18}
$$

Using the above equations, we obtain Eq. (14). The infinite series converges if $e^{-\lambda_e R}+e^{-\lambda_w R}<1$, otherwise it diverges. This leads to the expression of Eq. (8) for $E[D_1]_u$, proving the Lemma.

Next, we provide an expression for $E[D_2]_u$.

Lemma 4.2 *The expectation of time spent in phase 2 in the upper bound system $E[D_2]_u$ is given by Eq.* (15)*, where $\Pr(C_u)=1-\Pr\left(\bar{C}_u\right)$ is the probability that two consecutive eastbound nodes are connected.* $\Pr\left(\bar{C}_u\right)$ *is derived in Eq.* (18)*.*

Proof: In phase 2, nodes are connected and messages are able to propagate multihop. Phase 2 can effectively be divided in two parts. In the first part, the gap of N cells present during the previous phase 1 is bridged. Thus, the expected distance denoted by $E[D_{2,1}]$ traversed during this part is given by Eq. (16), where $\Pr\left(N=n|\bar{C}_u\right)$ is given by Eq. (12), and $\Pr\left(\bar{C}_u\right)$ is given by Eq. (18). Eq. (16) accounts for the fact that the next eastbound node is assumed to be located at the far-end extremity of the $(n+1)$-th cell, as per our upper bound construction. In the second part of phase 2, consecutive *eastbound* nodes remain connected as long as the distance between them is less than R, or, if the distance is greater than R, all *westbound* cells in the gap between the nodes are occupied. If the distance is greater than R, and not all *westbound* cells in the gap between the nodes are occupied, then the system re-enters phase 1 and the message is carried at vehicle speed. We note that it is possible that the distance traversed during the second part of phase 2 is zero.

Denote by C_u, the event that two consecutive nodes are connected and by $E[D'_{2,2}]_u$, the expected distance between two consecutive eastbound nodes, given that they are connected either directly or through westbound nodes. An expression for this quantity is the following:

$$
E[D'_{2,2}]_u = \int_0^{\infty} x f_{X_u|C_u}(x)dx, \tag{19}
$$

where $f_{X_u|C_u}(x)$ is the conditional distribution on the inter-vehicle distance based on the upper

$$
\begin{aligned}
E[D'_{2,2}]_u =& \int_0^\infty \frac{x f_{X_u}(x)\Pr(C_u|X_u=x)}{\Pr(C_u)}dx \\
=& \frac{1}{\Pr(C_u)}\left(\int_0^\infty \lambda_e e^{-\lambda_e x}(u(x)-u(x-R)) + \sum_{n=1}^{\infty}(1-e^{-\lambda_w R})^n \delta(x-(n+1)R)\right) x dx \\
=& \frac{1}{\Pr(C_u)}\left[\int_0^R x\lambda_e e^{-\lambda_e x}dx + \sum_{n=1}^{\infty}(n+1)R(1-e^{\lambda_w R})^n(e^{-\lambda_e nR}-e^{-\lambda_e(n+1)R})\right] \\
=& \frac{1}{\Pr(C_u)}\left[\frac{1}{\lambda_e}\left[1-e^{-\lambda_e R}(1+\lambda_e R)\right] + R(1-e^{-\lambda_e R})\left[\frac{e^{-\lambda_e R}(1-e^{-\lambda_w R})}{1-e^{-\lambda_e R}(1-e^{-\lambda_w R})} + \frac{e^{-\lambda_e R}(1-e^{-\lambda_w R})}{(1-e^{-\lambda_e R}(1-e^{-\lambda_w R}))^2}\right]\right].
\end{aligned}
\tag{20}
$$

bound distribution, given that nodes are connected. This conditional distribution can be computed as follows:

$$
f_{X_u|C_u}(x) = \frac{f_X(x)\Pr(C_u|X_u=x)}{\Pr(C_u)}, \tag{21}
$$

where $\Pr(C_u|X_u=x)$ denotes the probability that two consecutive eastbound nodes are connected for a given value of x. Nodes are always connected if the next eastbound node is within radio range, i.e. $x \le R$. If the inter-vehicle distance is greater than R, the nodes are connected if each of the corresponding n westbound cells are occupied, an event that occurs with probability $((1-e^{-\lambda_w R})^n)$.

Applying the upper bound distribution for inter-vehicle distance from Eq. (6):

$$
\Pr(C_u|X_u=x) = \begin{cases} 1 & \text{if } x \le R \\ (1-e^{-\lambda_w R})^n & \text{if } x=(n+1)R, \\ & \text{for } n=1,2,3,\ldots \\ 0 & \text{otherwise} \end{cases} \tag{22}
$$

Thus, the expected distance covered given that two consecutive eastbound nodes are connected is given by Eq. (20) where, from Eq. (18):

$$
\begin{aligned}
\Pr(C_u) =& 1-\Pr(\bar{C}_u) \\
=& (1-e^{-\lambda_e R})\left[1+\frac{e^{-\lambda_e R}(1-e^{-\lambda_w R})}{1-e^{-\lambda_e R}(1-e^{-\lambda_w R})}\right].
\end{aligned}
\tag{23}
$$

Once entering phase 2, messages propagate as long as connectivity is available, each time covering an expected distance of $E[D'_{2,2}]_u$ between two consecutive nodes. Hence, if connectivity is available for, say, j consecutive pairs of eastbound nodes, the distance covered is $jE[D'_{2,2}]_u$. Thus, the expected distance $E[D_{2,2}]$ covered during the second part of phase 2 is:

$$
\begin{aligned}
E[D_{2,2}]_u &= \sum_{j=1}^{\infty} jE[D'_{2,2}]\Pr(C_u)^j(1-\Pr(C_u)) \\
&= E[D'_{2,2}]_u(1-\Pr(C_u))\sum_{j=1}^{\infty} j\Pr(C_u)^j \\
&= E[D'_{2,2}]_u\frac{\Pr(C_u)}{(1-\Pr(C_u))}.
\end{aligned}
\tag{24}
$$

We finally obtain $E[D_2]_u = E[D_{2,1}]_u + E[D_{2,2}]_u$, leading to the expression given by the Lemma.

Based on the results of the previous Lemmas and Eq. (5), the next theorem provides an upper bound on v_{avg}.

Theorem 4.3 *The average message propagation speed is upper bounded as follows:*

$$v_{avg} \leq \begin{cases} \left(1 + \frac{E[D_2]_u}{E[D_1]_u}\right) v & \textit{if } e^{-\lambda_e R} + e^{-\lambda_w R} < 1 \\ v & \textit{if } e^{-\lambda_e R} + e^{-\lambda_w R} \geq 1, \end{cases}$$

where $E[D_1]_u$ and $E[D_2]_u$ are the expressions given by Lemmas 4.1 and 4.2.

Remark While our analysis is based on the assumption $v_{radio} = \infty$, Theorem 4.3 holds for any value of v_{radio} because v_{avg} is a non-decreasing function of v_{radio}.

4.5 Lower Bound Analysis

In the Appendix, we describe a lower bound on the average message propagation speed v_{avg}, based on the discretized system described in Section 4.2, i.e., assuming cells of size $R/2$ and an inter-node distance distribution as given by Eq. (7). We denote by $E[D_1]_l$ and $E[D_2]_l$ the expected distances traversed by a message in phase 1 and phase 2 during each cycle. The derivations of these quantities follow the same lines as the upper bound analysis. Once these quantities are computed, a lower bound on the average message propagation speed v_{avg} follows from Eq. (5).

Theorem 4.4 *Assume $v_{radio} = \infty$. The average message propagation speed is lower bounded as follows:*

$$v_{avg} \geq \begin{cases} \left(1 + \frac{E[D_2]_l}{E[D_1]_l}\right) v & \textit{if } e^{-\frac{\lambda_e R}{2}} + e^{-\frac{\lambda_w R}{2}} < 1 \\ v & \textit{if } e^{-\frac{\lambda_e R}{2}} + e^{-\frac{\lambda_w R}{2}} > 1, \end{cases}$$

where $E[D_1]_l$ and $E[D_2]_l$ are the expressions obtained from Lemmas A.1 and A.2, respectively.

4.6 Approximation

Based on the derivations for the upper bound and lower bound, one can provide an approximation model with the assumption that each cell is of size kR, where $0.5 < k < 1$. A reasonable value is $k = 0.75$.

Approximation 4.5 *The average message propagation speed for the approximation is:*

$$v_{avg} = \begin{cases} \frac{E[T_1]_a v + E[T_2]_a v_{radio}}{E[T_1]_a + E[T_2]_a} & \textit{if } e^{-\lambda_e k R} + e^{-\lambda_w k R} < 1 \\ v & \textit{if } e^{-\lambda_e k R} + e^{-\lambda_w k R} > 1, \end{cases}$$

where $E[T_1]_a$ and $E[T_2]_a$ are the approximations of the time spent in phase 1 and phase 2 respectively, obtained from equations (39) *and* (40) *in Lemma B.3 and B.4 respectively.*

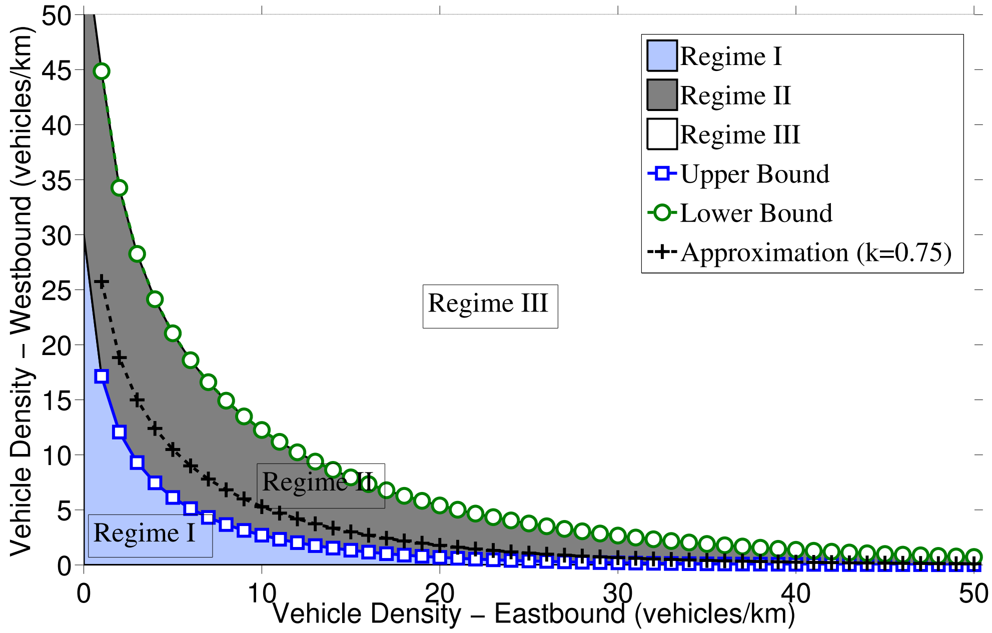


Figure 4: Three different regimes of message propagation speed, for $R = 125$ m. In `Regime I`, the average message propagation speed v_{avg} is the same as the vehicle speed v. In `Regime III`, v_{ave} is strictly larger than v and increases with the eastbound and westbound traffic densities λ_e and λ_w. The phase transition between these two regimes takes place somewhere in `Regime II`, as extrapolated by the approximation curve with $k = 0.75$.

4.7 Phase Transition

Theorems 4.3 and 4.4 provide upper and lower bounds on the average message propagation speed v_{avg}. Specifically, Theorem 4.3 reveals that if the combination of traffic densities in both directions is too low, i.e., $(e^{-\lambda_e R} + e^{-\lambda_w R}) > 1$, then v_{avg} does not exceed v, independently of the specific value of v. In this regime, Regime I, no gain is provided from the occasional opportunistic connectivity provided by the DTN architectures. On the other hand, Theorem 4.4 guarantees that if $(e^{-\frac{\lambda_e R}{2}} + e^{-\frac{\lambda_w R}{2}}) < 1$, Regime III, then the value of v_{avg} is strictly larger than v and increases with λ_e and λ_w. Thus, a phase transition takes place somewhere in the region of traffic densities $(e^{-\lambda_e R} + e^{-\lambda_w R}) < 1$ and $(e^{-\frac{\lambda_e R}{2}} + e^{-\frac{\lambda_w R}{2}}) > 1$, Regime II.

Figure 4 graphically shows the three different regimes for the case $R = 125m$. The figure shows that for low traffic density in one direction ($<$ 10 vehicles/km), a relatively high density of traffic in the other direction, ($10 - 25$ vehicles/km) is required. It is noteworthy, that in Regime I, a small increase in traffic density in either direction does not provide increase in the message propagation speed, as there are no gains to be achieved by the delay tolerant architecture. However, in Regime III, a small increase in density provides immediate gains in the message propagation speed.

The mathematical justification for the phase transition behavior is that, when the traffic density is too low, the expected distance to be traversed in phase 1 gets infinitely large. Looking back at Eq. (9) and our pattern matching problem analogy, we observe that the expected number of cells needed to bridge a certain gap N grows at a geometric rate with N, i.e., the growth rate is $1/(p_w) = 1/(1 - e^{-\lambda_w l})$, where l is the cell size ($l = R/2$ for the lower bound and $l = R$ for the upper bound). On the other hand, the inter-vehicle distance probability distribution decays at a geometric rate with N, i.e., the decay rate rate is $1 - p_e = e^{-\lambda_e l}$. Thus, for the expected distance in phase 1 to be finite, the product of these two rates must be smaller than one, since only in that case the infinite sum shown in Eq. (14) (for the upper bound) or Eq. (34) (for the lower bound) is finite. Thus, if $p_e + p_w < 1$, the average propagation speed is the same as the vehicle speed. On the other hand, if the density on either side of the roadway is high enough, such that $p_e + p_w > 1$, then a DTN messaging scheme becomes beneficial.

5 Performance Results

In this section, we evaluate the performance of delay tolerant network messaging with the help of both simulations and the analytical results derived in Section 4. Our goals are the following: 1) illustrate the phase transition phenomenon, through simulations for a realistic value of v_{radio}; 2) verify the accuracy of our approximation model; 3) verify the upper bound for finite v_{radio}; 4) use the approximation model to evaluate the impact of various parameters, such as vehicle density in each direction and vehicle speed, on the average message propagation speed performance; 5) compare the performance of DTN messaging with that of path establishing schemes.

The simulator, implemented in Matlab [31], follows the same model as described in Section 4.1, i.e., the distance between consecutive vehicles in each direction follows an i.i.d. exponential distribution. The simulations do not discretize the roadway as in the analysis and, thus, produce an estimate on the actual average message propagation speed. The simulation is repeated for 100 iterations, each iteration generating $10,000$ vehicles to account for the random node generation.

The system parameters are set as follows: radio speed $v_{radio} = 1000$ m/s, radio range $R = 125$ m, and vehicle speed $v = 20$ m/s (unless mentioned otherwise). The traffic density is varied from over a range of 1 vehicle/km to 100 vehicles/km, to cover the low, intermediate and high traffic density scenarios.

Average Message Propagation Speed

Results in Figure 5 depict the average message propagation speed for increasing vehicular traffic density. The traffic density is assumed to be numerically equivalent in both *eastbound* and *westbound* direction. We plot the *upper* bound and the approximation results derived in Section 4.

The simulation results are averaged over several iterations to account for random node generation and the resulting topology. The results clearly show the phase transition behavior. When the mean value of the vehicle traffic density is below 10 vehicles/km, the network is essentially disconnected and the messages are buffered within vehicles. The data traverse physical distance at vehicle speed ($v = 20$ m/s). When the node density is high (> 50 vehicles/km), the network is largely connected. Thus, data are able to propagate multihop through the network at the maximum speed permitted by the radio ($v_{radio} = 1000$ m/s). In medium node density, the network is comprised of disconnected sub-nets. There is transient connectivity in the network as vehicular traffic moves in opposing directions. As a result of the delay tolerant networking assumption and opportunistic forwarding, the message propagation alternates in the two phases. The average rate, a function of the time spent in each phase, is between the two extremes of v m/s and v_{radio} m/s.

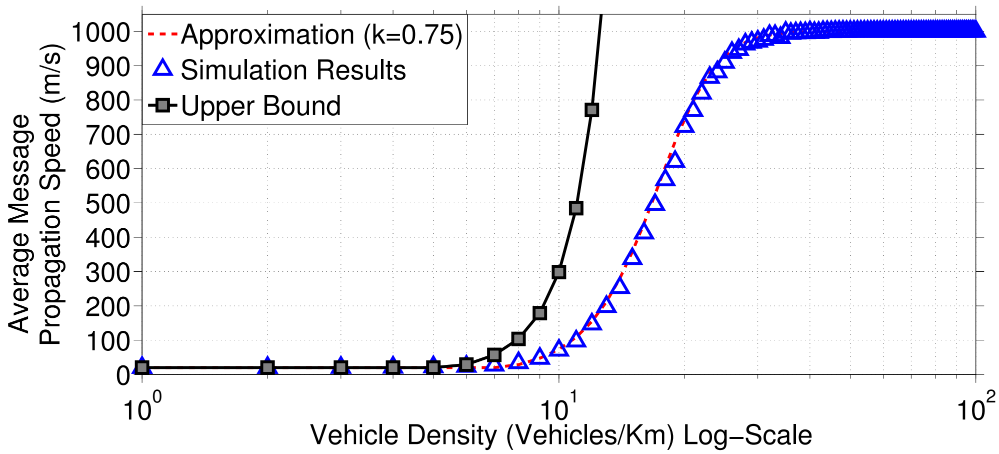


Figure 5: Comparison of simulation, analytical approximation, and upper bound on average message propagation speed as a function of traffic density.

Thus, the message propagation speed is a function of the connectivity in the network that is in turn determined by the vehicular traffic density for constant transmission range.

Figure 5 indicates that the analytical approximation derived in Section 4.6 is accurate, as the approximation closely follows the simulation results. As expected, the simulation curve lies below the *upper* bound. The bound is tight a low density, but diverges at high density since its derivation is based on the assumption $v_{radio} = \infty$.

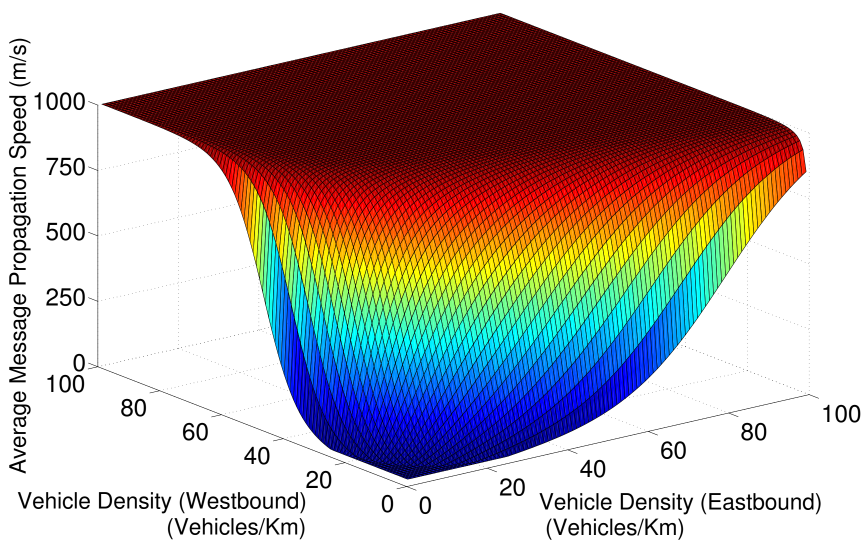


Figure 6: Average message propagation speed as a function of *eastbound* and *westbound* vehicular traffic densities, based on the approximation model.

In Fig. 6, we relax the assumption of symmetric values of traffic density along *eastbound* and *westbound* directions. We plot the average message propagation speed based on the approximation developed in Section 4.6 for values of *eastbound* and *westbound* traffic ranging from 1 vehicle/km to 100 vehicles/km. As is evident from the graph, the message rate increases as a function of the vehicular traffic density on either side of the roadway. The 3-dimensional graph allows us to map the message propagation speed for asymmetric values of traffic density on either side of the roadway. For example, if both *eastbound* and *westbound* directions have low traffic density of about 10 vehicles/km, then the node density is insufficient to enable message propagation. However, if the node density in the *eastbound* roadway is low, say 20 vehicles/km, while the *westbound* direction has higher traffic density, say 40 vehicles/km, then the node density is sufficient to reach the maximum performance of v_{radio} (1000 m/s).

Comparison with Path Establishing Routing Schemes

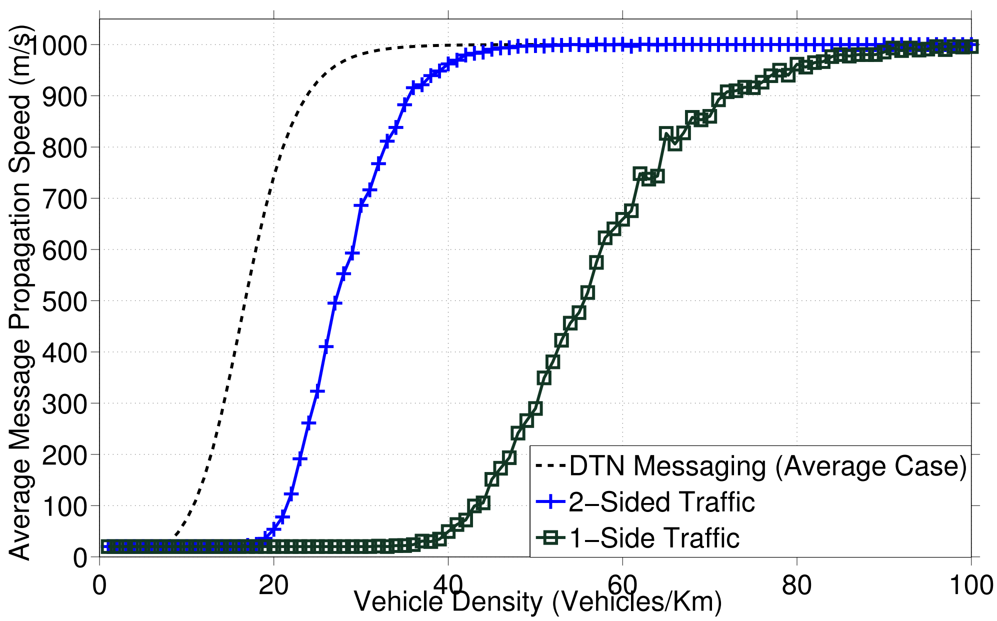


Figure 7: Comparison of DTN messaging strategy with path formation based schemes utilizing one-sided traffic or two-sided traffic for a distance of 12.5km.

In Fig. 7, we compare the average propagation speeds achievable for the approximation model of the delay tolerant architecture with that of a path establishing scheme, such as AODV or DSR. For the path establishing scheme, we assume that the destination of a message is fixed at a distance of 12.5 km from the source. The message propagates from the source through the network at multi-hop radio speed $v_{radio} = 1000$ m/s until it encounters a partition. Once a partition is encountered, the message is cached in a node's memory until the node reaches the destination goal of 12.5 km. The average message propagation speed is computed as the distance over the time taken to reach the destination. This result is averaged over several iterations. For one-sided traffic, only traffic along the eastbound direction is utilized in path formation. In the two-sided traffic model, nodes along both the eastbound and westbound direction are utilized in path formation. Thus, as a result, the scheme requires a high density of nodes for achieving end-to-end connectivity.

It is evident from Fig. 7 that a path establishing scheme that utilizes only one direction of traffic requires a density of at least 90 vehicles/km, on average, to achieve maximum performance. However, if vehicular nodes traveling in both directions are used for path formation, a density of about

$$E[D_1]_l = \begin{cases} \frac{R(1-e^{\lambda_e \frac{R}{2}})e^{-\frac{\lambda_e R}{2}}}{4\Pr(\bar{C}_l)e^{-\frac{\lambda_w R}{2}}} \left[\frac{e^{-\frac{\lambda_e R}{2}}}{1-e^{-\frac{\lambda_w R}{2}}-e^{-\frac{\lambda_e R}{2}}} + \frac{(1-e^{-\frac{\lambda_w R}{2}})e^{-\frac{\lambda_e R}{2}}}{1-e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})} - \frac{2e^{-\frac{\lambda_e R}{2}}}{1-e^{-\frac{\lambda_e R}{2}}} \right] & \text{if } e^{-\frac{\lambda_e R}{2}} + e^{-\frac{\lambda_w R}{2}} < 1 \\ \infty & \text{otherwise.} \end{cases} \tag{25}$$

45 vehicles/km is sufficient, on average. The DTN model achieves higher performance than both path establishing schemes for any given traffic density value.

Effect of Increased Mobility

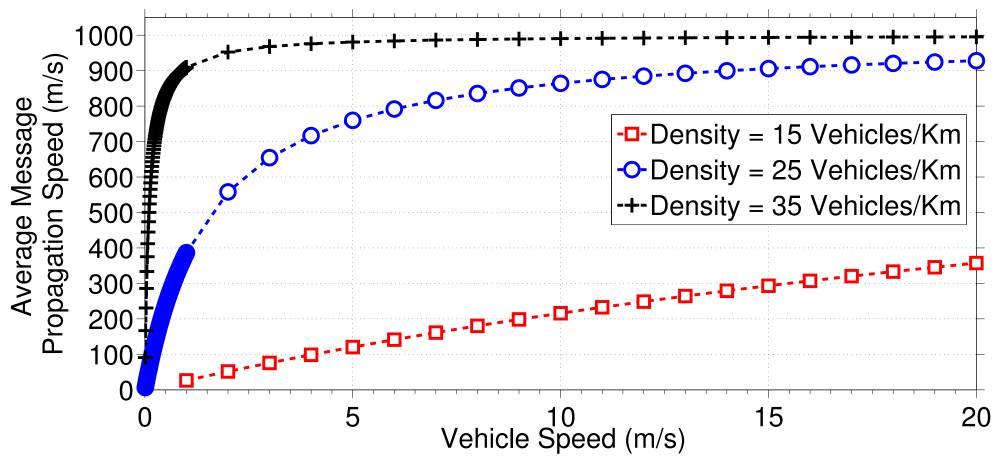


Figure 8: Impact of vehicle speed on average propagation speed for traffic densities, based on the approximation model.

In Fig. 8, we observe the performance of the messaging scheme as the vehicular speed increases at fixed values of *eastbound* and *westbound* traffic density. The graph shows that, for a vehicle density of 15 vehicles/km, the average message propagation speed increases from 0 m/s to 200 m/s as vehicular mobility increases from 0 m/s to 10 m/s. This is counter-intuitive to the observation in conventional MANET protocols that increased mobility decreases the messaging performance owing to short-lived paths. However, in this connection-less messaging paradigm, it is observed that the message exchange is aided by increased mobility. The partitions that occur in the network are bridged at a faster rate leading to increased performance.

6 Conclusion

In this paper, we characterize message propagation in a vehicular network with a delay tolerant networking (DTN) architecture. We propose a DTN-based routing scheme where vehicles traveling both in the same direction as the message and in opposing directions participate in the message forwarding. We develop an analytical model to model the routing scheme. The model takes into account the random distribution of distance between vehicles, the speed of vehicle, and radio parameters, such as the radio range. Based on the model, we derive an upper bound, lower bound and approximation on the average message propagation speed. Through simulation results, we show that the approximation model is accurate.

While the analysis relies on a discretized model, it does capture well the essence of the system behavior, namely the phase transition in the average message propagation speed as a function of the traffic density. The analysis reveals that the critical threshold of the phase transition depends only on the traffic density in each direction and on the radio range. Thus, through our analysis, we can identify the regimes of densities where the delay tolerant architecture is able or not to provide significant gains in messaging performance. We show that the messaging performance predominantly lies in between two extremes. For sufficiently high traffic density, the network behaves as if it were fully connected and the maximum speed of messaging is achieved. At the other extreme, for low traffic density, the network is mostly partitioned and no gains from delay tolerant architecture are achievable. These results imply that DTN-based VANET architectures prove most useful at medium traffic densities. (e.g., 20 vehicles/km) and higher. Furthermore, our simulations show the superiority of DTN-based routing schemes over those based on path establishment, such as AODV and DSR. In the former case, maximum performance is achieved with traffic densities as low as 20 vehicles/km, while the latter schemes require densities of 45 vehicles/km or higher. These numbers are based on the assumption of a transmission range $R = 125$ m. If the value of R changes, then the corresponding values for the traffic density will change accordingly.

This paper can serve as the basis for several interesting extensions. For instance, our model assumes that all the vehicles travel at the same speed. As a result, a phase transition is observed only because of two-sided traffic (i.e., there would be no phase transition with traffic present in only one direction). It would be interesting to investigate whether or not the same conclusion holds if vehicles move at different speeds. Similarly, the issue of multi-lane highways with speed differentials across the lanes is an interesting area open for further research.

7 Appendix

(a) Lower Bound Analysis

We derive a lower bound on the average message propagation v_{avg}. We denote by $E[D_1]_l$ and $E[D_2]_l$ the expected distance traversed in phase 1 and phase 2, respectively, during each cycle. The following Lemma provides an expression for $E[D_1]_l$.

Lemma A.1 *The expectation of the expected distance traversed in phase 1 in the lower bound system is given by Eq.* (25)*, where* $\Pr(\bar{C}_l)$ *is the probability that nodes are disconnected, an expression for which is given by Eq.* (33)*.*

Proof: The expected distance traversed between two consecutive eastbound nodes in phase 1, given a gap of $N = n$ cells between them is is given by:

$$E[D_1|N=n]_l = \frac{R}{4}\left[\frac{1-(1-e^{-\frac{\lambda_w R}{2}})^n}{e^{-\frac{\lambda_w R}{2}}(1-e^{-\frac{\lambda_w R}{2}})^n}\right]. \tag{26}$$

Note that we did not subtract n within this equation. The reason is that, for the lower bound, we must account for the fact that one of the first n cells must be empty (otherwise, the nodes would

$$E[D_2]_l = \frac{R(1-e^{-\frac{\lambda_e R}{2}})e^{-\frac{\lambda_e R}{2}}}{2\Pr(\bar{C}_l)} \left[\frac{e^{-\frac{\lambda_e R}{2}}}{(1-e^{-\frac{\lambda_e R}{2}})} + \frac{e^{-\frac{\lambda_e R}{2}}}{(1-e^{-\frac{\lambda_e R}{2}})^2} - \frac{e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})}{1-e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})} - \frac{e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})}{(1-e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}}))^2} \right]$$
$$+ \frac{1}{\Pr(\bar{C}_l)} \left[\frac{1}{\lambda_e} \left[1 - e^{-\lambda_e R}(1+\lambda_e R) \right] \right.$$
$$\left. + \frac{R}{2}(1-e^{-\frac{\lambda_e R}{2}})e^{-\frac{\lambda_e R}{2}} \left[\frac{e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})}{1-e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})} + \frac{e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})}{(1-e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}}))^2} \right] \right]. \tag{28}$$

$$E[D'_{2,2}]_l = \int_0^\infty \frac{x f_{X_l}(x)\Pr(C_l|X_l = x)}{\Pr(C_l)} dx$$
$$= \frac{1}{\Pr(C_l)} \left[\frac{1}{\lambda_e} \left[1 - e^{-\lambda_e R}(1+\lambda_e R) \right] + \frac{R}{2}(1-e^{-\frac{\lambda_e R}{2}})e^{-\frac{\lambda_e R}{2}} \left[\frac{e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})}{1-e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})} + \frac{e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})}{(1-e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}}))^2} \right] \right], \tag{29}$$

---

have been connected). Hence, we conservatively add n cells to the distance traversed in phase 1, which means that a message spends a relatively larger fraction of its time in phase 1 traveling at vehicle speed v.

Denote by $\bar{C}_l$, the event that two consecutive eastbound nodes are disconnected. Then,

$$E[D_1]_l = \sum_{n=1}^{\infty} E[D_1|N=n]_l \Pr(N=n|\bar{C}_l). \tag{27}$$

We again compute $\Pr(N=n|\bar{C}_l)$ using Bayes' Law, i.e.:

$$\Pr(N=n|\bar{C}_l) = \frac{\Pr(\bar{C}_l|N=n)\Pr(N=n)}{\Pr(\bar{C}_l)}. \tag{30}$$

We have:

$$\Pr(\bar{C}_l|N=n) = 1-(1-e^{-\frac{\lambda_w R}{2}})^n; \tag{31}$$
$$\Pr(N=n) = (e^{-\lambda_e(n+1)\frac{R}{2}} - e^{-\lambda_e(n+2)\frac{R}{2}}); \tag{32}$$

$$\Pr(\bar{C}_l) = \sum_{n=1}^{\infty} \Pr(\bar{C}_l|N=n)\Pr(N=n)$$
$$= e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_e R}{2}}) \left[\frac{e^{-\frac{\lambda_e R}{2}}}{1-e^{-\frac{\lambda_e R}{2}}} \right.$$
$$\left. - \frac{e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})}{1-e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})} \right]. \tag{33}$$

Using the above equations, we obtain:

$$
\begin{aligned}
E[D_1]_l &= \sum_{n=1}^{\infty} E[D_1|N=n]_l \Pr(N=n|\bar{C}_l) \\
&= \frac{R(1-e^{\lambda_e \frac{R}{2}})e^{\lambda_e \frac{R}{2}}}{4\Pr(\bar{C}_l)e^{-\frac{\lambda_w R}{2}}} \left[\frac{e^{-\frac{\lambda_e R}{2}}}{1-e^{-\frac{\lambda_w R}{2}}-e^{-\frac{\lambda_e R}{2}}} \right. \\
&\left. + \frac{e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})}{1-e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})} - \frac{2e^{-\frac{\lambda_e R}{2}}}{1-e^{-\frac{\lambda_e R}{2}}} \right].
\end{aligned} \tag{34}
$$

We note that the above expression holds only if $e^{-\frac{\lambda_e R}{2}} + e^{-\frac{\lambda_w R}{2}} < 1$, otherwise the series is divergent, leading to the expression provided by the Lemma.

Next, we provide an expression for $E[D_2]_l$.

Lemma A.2 *The expectation of the distance traversed in phase 2 in the lower bound system is given by Eq.* (28)*, where* $\Pr(C_l) = 1 - \Pr(\bar{C}_l)$ *and* $\Pr(\bar{C}_l)$ *is given by Eq.* (33)*.*

Proof: The expected distance denoted $E[D_{2,1}]$ traversed during the first part of phase 2 is given by Eq. (35).

$$
\begin{aligned}
E[D_{2,1}]_l &= \frac{R}{2}\sum_{n=1}^{\infty}(n+1)\Pr(N=n|\bar{C}_l) \\
&= \frac{R(1-e^{-\frac{\lambda_e R}{2}})e^{-\frac{\lambda_e R}{2}}}{2\Pr(\bar{C}_l)} \left[\frac{e^{-\frac{\lambda_e R}{2}}}{(1-e^{-\frac{\lambda_e R}{2}})} \right. \\
&+ \frac{e^{-\frac{\lambda_e R}{2}}}{(1-e^{-\frac{\lambda_e R}{2}})^2} - \frac{e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})}{1-e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})} \\
&\left. - \frac{e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}})}{(1-e^{-\frac{\lambda_e R}{2}}(1-e^{-\frac{\lambda_w R}{2}}))^2} \right].
\end{aligned} \tag{35}
$$

where $\Pr(N=n|\bar{C}_l)$ is given by Eq. (30), and $\Pr(\bar{C}_l)$ is given by Eq. (33). Denote by $E[D'_{2,2}]_l$ the expected distance between two consecutive eastbound nodes, given that they are connected either directly or through westbound nodes. An expression for this quantity is the following:

$$
E[D'_{2,2}]_l = \int_0^{\infty} x f_{X_l|C_l}(x)dx, \tag{36}
$$

where $f_{X_l|C_l}(x)$ is the conditional distribution on the inter-vehicle distance, based on the lower bound distribution, given that nodes are connected. This distribution is computed as:

$$
f_{X_l|C_l}(x) = \frac{f_X(x)\Pr(C_l|X_l=x)}{\Pr(C_l)}, \tag{37}
$$

$$E[T_1]_a = \begin{cases} \frac{kR(1-e^{\lambda_e kR})e^{-\lambda_e kR}}{2v\Pr(\bar{C}_a)} \left[\frac{1}{e^{-\lambda_w kR}} \left\{ \frac{e^{-\lambda_e kR}}{1-e^{-\lambda_w kR}-e^{-\lambda_e kR}} + \frac{e^{-\lambda_e kR}(1-e^{-\lambda_w kR})}{1-e^{-\lambda_e kR}(1-e^{-\lambda_w kR})} \right.\right. & \\ \left.\left. -\frac{2e^{-\lambda_e kR}}{1-e^{-\lambda_e kR}} \right\} - \left\{ \frac{e^{-\lambda_e kR}}{(1-e^{-\lambda_e kR})^2} - \frac{e^{-\lambda_e kR}(1-e^{-\lambda_w kR})}{(1-e^{-\lambda_e kR}(1-e^{-\lambda_w kR}))^2} \right\} \right] & \text{if } e^{-\lambda_e kR} + e^{-\lambda_w kR} < 1 \\ \infty & \text{otherwise.} \end{cases} \tag{39}$$

$$\begin{aligned} E[T_2]_a = {} & \frac{kR(1-e^{-\lambda_e kR})}{v_{radio}\Pr(\bar{C}_a)} \left[\frac{e^{-\lambda_e kR}}{(1-e^{-\lambda_e kR})} + \frac{e^{-\lambda_e kR}}{(1-e^{-\lambda_e kR})^2} - \frac{e^{-\lambda_e kR}(1-e^{-\lambda_w kR})}{1-e^{-\lambda_e kR}(1-e^{-\lambda_w kR})} - \frac{e^{-\lambda_e kR}(1-e^{-\lambda_w kR})}{(1-e^{-\lambda_e kR}(1-e^{-\lambda_w kR}))^2} \right] \\ & + \frac{1}{v_{radio}\Pr \bar{C}_a} \left[\frac{1}{\lambda_e} \left[1 - e^{-\lambda_e R}(1+\lambda_e kR) \right] \right. \\ & \left. + kR(1-e^{-\lambda_e kR}) \left[\frac{e^{-\lambda_e kR}(1-e^{-\lambda_w kR})}{1-e^{-\lambda_e kR}(1-e^{-\lambda_w kR})} + \frac{e^{-\lambda_e kR}(1-e^{-\lambda_w kR})}{(1-e^{-\lambda_e kR}(1-e^{-\lambda_w kR}))^2} \right] \right]. \end{aligned} \tag{40}$$

---

where $\Pr(C_l|X_l = x)$ denotes the probability the nodes are connected for a given value of x, given by:

$$\Pr(C_l|X_l = x) = \begin{cases} 1 & \text{if } x \leq R \\ (1 - e^{-\lambda_w (n+1)\frac{R}{2}}) & \text{if } x = (n+1)\frac{R}{2}, \\ & \text{for } n = 1, 2, 3, \ldots \\ 0 & \text{otherwise.} \end{cases} \tag{38}$$

Applying the lower bound distribution for inter-vehicle distance from Eq. (7), we obtain Eq. (29), where $\Pr(C_l) = 1 - \Pr(\bar{C}_l)$. In phase 2, the distance $E[D'_{2,2}]_l$ is the expected distance covered between two consecutive nodes. Thus, the expected distance $E[D_{2,2}]$ covered during second part of phase 2 is:

$$\begin{aligned} E[D_{2,2}]_l &= \sum_{j=1}^{\infty} jE[D'_{2,2}] \Pr(C_l)^j (1 - \Pr(C_l)) \\ &= E[D'_{2,2}]_l \frac{\Pr(C_l)}{(1 - \Pr(C_l))}. \end{aligned} \tag{41}$$

We finally obtain $E[D_2]_l = E[D_{2,1}]_l + E[D_{2,2}]$, leading to the expression given by the Lemma.

(b) Approximation

Approximation B.3 *An approximation of the expected time spent in phase 1 $E[T_1]_a$ is given by equation* (39)*, where $\Pr(\bar{C}_a)$ is the probability nodes are disconnected, given by:*

$$\begin{aligned} \Pr(\bar{C}_a) &= \sum_{n=1}^{\infty} \Pr(\bar{C}_a|N = n) \Pr(N = n) \\ &= (1 - e^{-\lambda_e kR}) e^{-\lambda_e kR} \left[\frac{e^{-\lambda_e kR}}{1 - e^{-\lambda_e kR}} \right. \\ &\quad \left. - \frac{e^{-\lambda_e kR}(1 - e^{-\lambda_w kR})}{1 - e^{-\lambda_e kR}(1 - e^{-\lambda_w kR})} \right]. \end{aligned} \tag{42}$$

Approximation B.4 *An approximation of time spent is phase 2, $E[T_2]_a$, is given by the expression in Eq.* (40)*, where* $\Pr(\bar{C}_a)$ *is the probability that nodes are disconnected given by Eq.* (42)*. For detailed derivations of the approximation model, we refer to [32].*

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